Technical Comments

Comment on "Improved Thin-Airfoil Theory"

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IN Ref. 1, the authors claim to extend the range of applicability of classical thin-airfoil theory by including the effect of profile thickness on the aerodynamic lift and moment. Classical (or first order) thin-airfoil theory is a small-disturbance theory that keeps terms linear in airfoil thickness ratio, camber ratio, and angle of attack, and correctly demonstrates that thickness influences the lift and pitching moment only at second order. It is the main purpose of this Technical Comment to correctly use thin-airfoil theory (as extended to second order by Van Dyke[2]) to find the effect of thickness on lift and to demonstrate that the results presented in Ref. 1 are an inconsistent use of the theory.

The analysis that follows is a slightly modified version of the second-order thin-airfoil theory presented by Van Dyke.[2] Let the airfoil surface be given by

$$y = Y(x) = C(x) \pm T(x) \qquad -c/2 \le x \le c/2 \tag{1}$$

where the x axis is along the chord line with origin at midchord, c is the chord length, $C(x)$ is the mean camber line, and $T(x)$ is the thickness distribution. The uniform stream has speed V_∞ and is at an angle of attack α. The velocity components U and V are expanded in series in powers of the thickness ratio, camber ratio, and angle of attack, and terms up to second order are kept. The velocity components are then

$$U = V_\infty(1-\alpha^2/2) + u_1 + u_2 \tag{2a}$$

$$V = V_\infty\alpha + v_1 + v_2 \tag{2b}$$

where the subscript 1 represents first order and the subscript 2 represents second order.

First-Order Problem—Classical Thin-Airfoil Theory

The airfoil boundary condition, which has been transferred to the chordline, is

$$\frac{v_1(x,0\pm)}{V_\infty} = Y'(x) - \alpha = C'(x) \pm T'(x) - \alpha \tag{3}$$

On the chordline, the solution for u_1 consists of a thickness term

$$\frac{u_{1T}}{V_\infty} = \frac{1}{\pi}\int_{-c/2}^{c/2} \frac{T'(x_0)\,\mathrm{d}x_0}{x-x_0} \tag{4}$$

and a term due to camber and angle of attack

$$\frac{u_{1L}}{V_\infty} = \left(\frac{c/2-x}{x+c/2}\right)^{1/2}\left[\alpha + \frac{1}{\pi}\int_{-c/2}^{c/2}\left(\frac{x_0+c/2}{c/2-x_0}\right)^{1/2}\frac{C'(x_0)\,\mathrm{d}x_0}{x-x_0}\right] \tag{5}$$

Note that the integrals in Eqs. (4) and (5) are Cauchy principal value integrals. The surface speed is then given to first order as

$$\frac{q_1}{V_\infty} = 1 + \frac{u_{1T}}{V_\infty} \pm \frac{u_{1L}}{V_\infty} \tag{6}$$

and the first-order pressure coefficient is

$$C_{p1} = -\frac{2}{V_\infty}(u_{1T} \pm u_{1L}) \tag{7}$$

The airfoil lift coefficient is then determined from

$$C_L = \frac{1}{c}\int_{-c/2}^{c/2}\Big[C_p(x,0-) - C_p(x,0+)\Big]\,\mathrm{d}x \tag{8}$$

and the first-order lift coefficient is

$$C_L = \frac{4}{V_\infty c}\int_{-c/2}^{c/2} u_{1L}\,\mathrm{d}x \tag{9}$$

Note that the lift coefficient in Eq. (9) is independent of thickness.

Second-Order Problem

The airfoil boundary condition can be written as

$$\frac{v_2(x,0\pm)}{V_\infty} = C_2'(x) \pm T_2'(x) \tag{10}$$

where the fictitious camber and thickness functions are

$$C_2(x) = \frac{u_{1T}}{V_\infty}C + \frac{u_{1L}}{V_\infty}T \tag{11a}$$

$$T_2(x) = \frac{u_{1T}}{V_\infty}T + \frac{u_{1L}}{V_\infty}C \tag{11b}$$

The second-order problem is almost identical to the first-order one, and on the chordline the contribution to u_2 from the T_2 term is

$$\frac{u_{2T}}{V_\infty} = \frac{1}{\pi}\int_{-c/2}^{c/2}\frac{T_2'(x_0)\,\mathrm{d}x_0}{x-x_0} \tag{12}$$

and the contribution from the C_2 term is

$$\frac{u_{2L}}{V_\infty} = \frac{1}{\pi}\left(\frac{c/2-x}{x+c/2}\right)^{1/2}\int_{-c/2}^{c/2}\left(\frac{x_0+c/2}{c/2-x_0}\right)^{1/2}\frac{C_2'(x_0)\,\mathrm{d}x_0}{x-x_0} \tag{13}$$

The surface speed at second order also contains a contribution from the boundary condition transfer from the surface to the

Received March 4, 1989.

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chordline and is

$$\frac{q_2}{V_\infty} = 1 + \frac{u_{1T}}{V_\infty} \pm \frac{u_{1L}}{V_\infty} + \frac{u_{2T}}{V_\infty} \pm \frac{u_{2L}}{V_\infty} + (C \pm T)(C'' \pm T'') + \frac{1}{2}(C' \pm T')^2 - \frac{\alpha^2}{2} \tag{14}$$

and the surface pressure coefficient is

$$C_{p2} = -2\left(\frac{q_2}{V_\infty} - 1\right) - \left(\frac{u_{1T}}{V_\infty} \pm \frac{u_{1L}}{V_\infty}\right)^2 \tag{15}$$

The lift coefficient (correct to second order) is obtained from the substitution of Eq. (15) into Eq. (8) and is

$$C_L = \frac{4}{c}\int_{-c/2}^{c/2}\left\{\frac{u_{1L}}{V_\infty} + \frac{u_{1T}u_{1L}}{V_\infty^2} + \frac{u_{2L}}{V_\infty} + TC'' + CT'' + C'T'\right\}\mathrm{d}x \tag{16}$$

Using the terminology of this Comment, the lift coefficient for the improved thin-airfoil theory of Ref. 1 is

$$C_L = \frac{4}{c}\int_{-c/2}^{c/2}\left(\frac{u_{1L}}{V_\infty} + \frac{u_{1T}u_{1L}}{V_\infty^2}\right)\mathrm{d}x \tag{17}$$

and it can be seen that it is missing the last four terms in the integrand of the correct result in Eq. (16). The lift coefficient in Eq. (17) was obtained by keeping the quadratic (second-order) term in Bernoulli's equation for the pressure coefficient [for use in Eq. (8)] even though terms comparable in magnitude had been neglected in the determination of the velocity components.

In addition to the above incorrect use of thin-airfoil theory to include the effect of thickness in the lift and moment coefficients, the authors present a technique to evaluate the Fourier coefficients for the velocity using the airfoil ordinates rather than the airfoil slopes. For the integral equation version of thin-airfoil theory presented here, a similar technique is given in Van Dyke[2] that is based on results published by Riegels and Wittich in the 1940s.

Also, the results of the analysis of Ref. 1 for a series of Kármán-Trefftz airfoils are compared with exact solutions using complex variables as well as the results of a panel code identified as the Hess-Smith method adopted by Moran.[3] It is claimed that the improved thin-airfoil theory (with a leading-edge correction for the pressure) outperforms the panel method for a certain range of the airfoil parameters.

According to Hess,[4] the code used in Ref. 1 is the 1967 version. The Kármán-Trefftz airfoils for which the panel method lift are most in error are limiting cases of the airfoil shape having sharp leading edges (for example, $\beta = 9$ deg, $\tau = 0.04$, $\epsilon = 0$ and $\beta = 13.5$ deg, $\tau = 0.06$, $\epsilon = 0.04$) and therefore not appropriate for comparison. Hess states that "the current higher-order panel method has no trouble with such shapes." He also states that "the difficulty the 1967 code has at cusped trailing edges is well-known" and was resolved in Hess.[5] "To remove the pressure crossing near the trailing edge, recourse to the higher-order method is not necessary, but it suffices to use the lower-order methods with a surface vorticity weighted parabolically (instead of constant) around the airfoil in such a way that it falls to zero at the upper and lower trailing edge."

Finally, a word needs to be said about the pressure distribution results presented by the authors. The results of thin-airfoil theory are invalid in the neighborhood of stagnation points that occur near round leading edges or at finite angle trailing edges. The authors introduce a correction due to Riegels to alleviate the "unrealistically high velocities near the leading edge" but then claim that the results for a very thin airfoil (shown in Fig. 4) are "almost identical" with or without the correction. The extent of the neighborhood of the leading edge where the thin-airfoil results are invalid is of the order of the thickness ratio squared and an expanded plot of this region in Fig. 4 would clearly show the same behavior seen in Fig. 5. An excellent discussion of the edge singularities of thin-airfoil theory and a unified theory of corresponding leading-edge corrections can be found in Van Dyke.[2]

In conclusion, it appears that the "improvements" to thin-airfoil theory that the authors present have been provided correctly in 1956 by Van Dyke[2] and that accurate airfoil solutions can be obtained from the current versions of many available panel codes.

References

[1]Zedan, M. F., and Abu-Abdou, K., "Improved Thin-Airfoil Theory," *Journal of Aircraft*, Vol. 25, No. 12, 1988, pp. 1122–1128.

[2]Van Dyke, M. D., "Second-Order Subsonic Airfoil Theory Including Edge Effects," NACA Rept. 1274, 1956, pp. 541–560.

[3]Moran, J., *An Introduction to Theoretical and Computational Aerodynamics*, Wiley, New York, 1984.

[4]Hess, J. L., Private communication, letter dated Feb. 14, 1989.

[5]Hess, J. L., "The Use of Higher-Order Surface Singularity Distributions to Obtain Improved Potential Flow Solutions for Two-Dimensional Lifting Airfoils," *Computer Methods in Applied Mechanics and Engineering*, Vol. 5, No. 1, 1975, pp. 11–35.

Reply by Authors to A. Plotkin

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THE authors wish to thank A. Plotkin for his interest in their work. The technical comment addresses three points: 1) the order of accuracy of the method, 2) comparison with the panel method, and 3) the results of the parametric study. In response to the first point, we would like to emphasize that the method presented in the paper was not intended or claimed to be a second-order, thin-airfoil theory at all; in fact, nothing was mentioned about the order of the method. Although we are glad to see that our expression for C_L shares some common terms with a more rigorous second-order theory, we believe that this part of the technical comment has missed the main objective of the work. It was repeatedly stressed that the power of the improved thin-airfoil theory lies in the fact that enhanced accuracy is achieved while fully maintaining the main popular characteristics of the classical theory; namely, closed-form solution, simplicity, and ease of programming. The presented method may be considered as an improved first-order, thin-airfoil theory if one wishes to do so. The improvement is brought about by making no approximations in evaluating the pressure distribution beyond those already made in obtaining the velocity expression in Eq. (6) of the comment. There is no obvious reason, at least to us, to apply the linearized Eq. (7) to obtain the pressure coefficient; instead we have directly applied the Bernoulli equation. We do not see how the use of an exact relation like the Bernoulli equation could be disqualified as an "incor-

Received April 26, 1989.

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